Stats 401 Lab 6

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10/12/2018

Lab Activity (Part 1)

1. If $\text{Cor}(W, Z) = 0.5$, what is the correlation of $\text{Cor}(2W, Z + 1)$?
2. Let (X, Y) take the values $(0, 1), (1, 1), (1, 2)$, each with probability $1/3$
   - What is the covariance of X and Y?
   - We take a sample of size 5: $(0, 1), (0, 1), (1, 2), (1, 1), (1, 2)$. What is sample covariance?

Lab Activity (Part 1) Solutions

Question 1 Part 1

$$Cor(2W, Z+1) = \frac{Cov(2W, Z+1)}{\sqrt{var(2W)var(Z+1)}}$$

$$\begin{aligned} Cov(2W, Z+1) &= E[2W - E(2W)]E[(Z+1) - E(Z+1)] \\ &= E[2W - 2E(W)]E[Z+1-(E(Z)+1)] \\ &= 2E[W - E(W)]E[Z - E(Z)] \\ &= 2Cov(W, Z) \end{aligned}$$

Lab Activity (Part 1) Solutions, cont.

Question 1 Part 1 (cont)

$$Var(2W) = 4Var(W)$$
$$Var(Z+1) = Var(Z)$$

$$Cor(2W, Z+1) = \frac{2Cov(W,Z)}{\sqrt{4Var(W)Var(Z)}}$$

$$= \frac{Cov(W,Z)}{\sqrt{Var(W)Var(Z)}}$$

$$= Cor(W,Z) = 0.5$$

Lab Activity (Part 1) Solutions, cont.

Question 1 Part 2

$$cov(x, y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{4-1}\sum_{i=1}^{4}(x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{4}\sum_{i=1}^{4}(x_i - \bar{x})(y_i - \bar{y})$$

$$\bar{x} = \frac{3}{5}$$

$$\bar{y} = \frac{7}{5}$$

Lab Activity (Part 1) Solutions, cont.

Question 1 Part 2

$$
\begin{aligned}
cov(x,y) &= \frac{1}{4}\sum_{i=1}^{4}(x_i - \frac{3}{5})(y_i - \frac{7}{5}) \\
&= \frac{1}{4}[(0-\frac{3}{5})(1-\frac{7}{5}) + (0-\frac{3}{5})(1-\frac{7}{5}) + \\
&\quad (1-\frac{3}{5})(2-\frac{7}{5}) + (1-\frac{3}{5})(1-\frac{7}{5}) + \\
&\quad (1-\frac{3}{5})(2-\frac{7}{5})] \\
&= \frac{1}{4}[(-\frac{3}{5})(-\frac{2}{5}) + (-\frac{3}{5})(-\frac{2}{5}) + \\
&\quad (\frac{2}{5})(\frac{3}{5}) + (\frac{2}{5})(-\frac{2}{5}) + \\
&\quad (\frac{2}{5})(\frac{3}{5})]
\end{aligned}
$$

Lab Activity (Part 1) Solutions, cont.

Question 1 Part 2 cont.

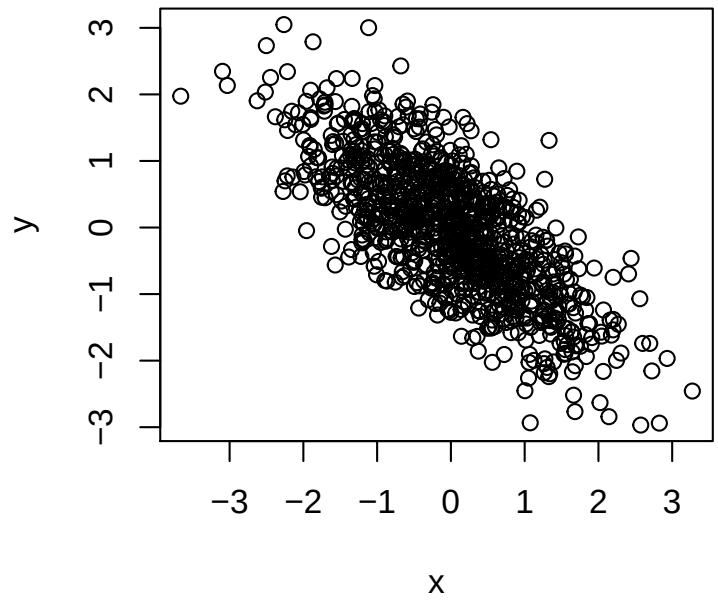
$$= \frac{1}{4}[\frac{6}{25} + \frac{6}{25} + \frac{6}{25} - \frac{4}{25} + \frac{6}{25}]$$

$$= \frac{1}{4} \times \frac{20}{25}$$

$$= \frac{5}{25} = \frac{1}{5}$$

Lab Activity (Part 2)

The scatterplot below was generated from a bivariate normal distribution with mean vector $(0, 0)$

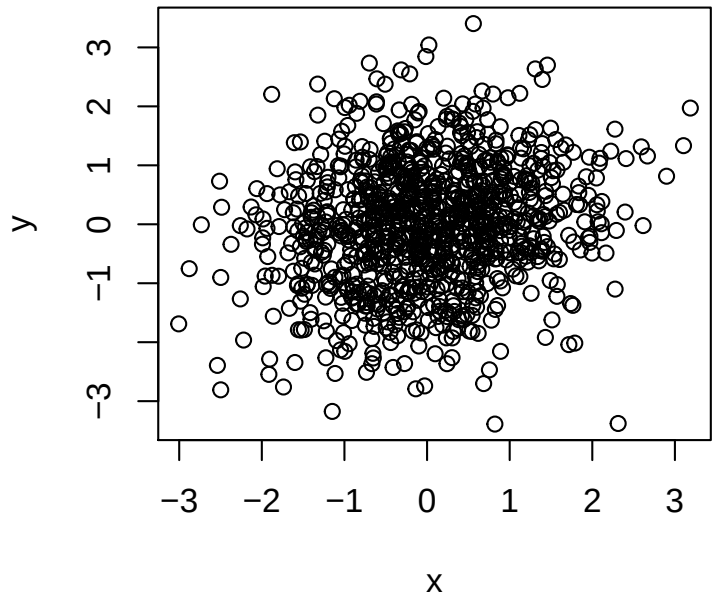


Which of the following is the variance-covariance matrix?

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; 2. $\begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix}$; **3.** $\begin{bmatrix} 1 & -0.75 \\ -0.75 & 1 \end{bmatrix}$

Lab Activity (Part 2)

The scatterplot below was generated from a bivariate normal distribution with mean vector $(0, 0)$

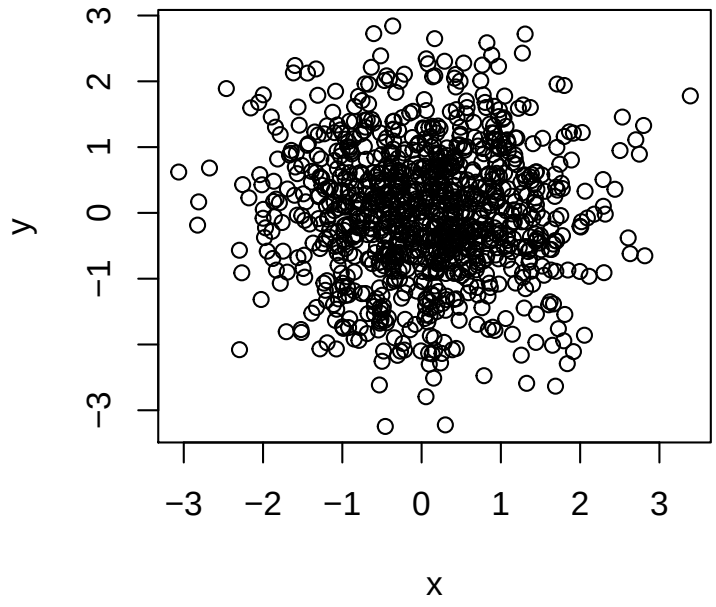


Which of the following is the variance-covariance matrix?

1. $\begin{bmatrix} 1 & -0.2 \\ -0.2 & 1 \end{bmatrix}$; **2.** $\begin{bmatrix} 1 & 0.2 \\ 0.2 & 1 \end{bmatrix}$; 3. $\begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix}$

Lab Activity (Part 2)

The scatterplot below was generated from a bivariate normal distribution with mean vector $(0, 0)$



Which of the following is the variance-covariance matrix?

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; 2. $\begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix}$; 3. $\begin{bmatrix} 1 & -0.75 \\ -0.75 & 1 \end{bmatrix}$

Lab Ticket

1. Why is $\begin{bmatrix} 4 & 0 \\ 0.25 & 4 \end{bmatrix}$ not a valid variance-covariance matrix?
2. Let (X, Y) be bivariate normal with mean $(6, 4)$ and variance-covariance matrix $\mathbb{V} = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$.
   - What are the mean and standard deviation of Y?
   - What is the covariance of X and Y?

Lab Ticket Solutions

1. It is not a valid variance-covariance because it is not symmetric.
2. (a.) The mean of Y is 4 and the standard deviation of Y is 3.
3. (b.) The covariance of X and Y is 0; this does not mean that they are independent.