Lab 8 solutions

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Exit ticket solution: Q1

$$
\begin{aligned}
-1 &\leq \mathrm{Cor}(X, Y) \leq 1 \\
\implies -1 &\leq \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}} \leq 1 \\
\implies -1 &\leq \frac{\mathrm{Cov}(X, Y)}{\sqrt{(6)(6)}} \leq 1 \\
\implies -6 &\leq \mathrm{Cov}(X, Y) \leq 6
\end{aligned}
$$

Exit ticket solution: Q2

- Since X is normal, $2X + 1$ is also normal.
  - $\mathrm{E}(2X + 1) = 2\mathrm{E}(X) + 1 = 3$
  - $\mathrm{Var}(2X + 1) = 2^2\mathrm{Var}(X) = 4(4) = 16$

Hence, $X \sim \mathcal{N}(3, 16)$

- Since (X, Y) is bivariate normal and X, Y are independent, hence $\mathrm{Cov}(X, Y) = 0$
  - $\mathrm{E}(X - 2Y) = \mathrm{E}(X) - 2\mathrm{E}(Y) = 1 - 4 = 3$
  - $\mathrm{Var}(X - 2Y) = \mathrm{Var}(X) + (-2)^2\mathrm{Var}(Y) = 4 + 4(9) = 40$

Hence, $X - 2Y \sim \mathcal{N}(3, 40)$