Stats 401 Lab 2

401 GSI team

1/9/2018

GSI Office hour

At 2165 USB, the Science Learning Center Annex

- Yuan Sun. Mon 9-11 am
- Sanjana Gupta. Tue 9-11 am
- Naomi Giertych. Thu 9-11 am

Homework

- Out of 10 points
- 1 point for the statement of sources
- 1 point for feedback
- Provide the code if the question request

Swirl tutorial

We have finished lesson 1/3/4 in HW1.

- Any techical difficulties encountered working with swirl?
- Any questions about materials introduced in the tutorial?

Swirl tutorial

You are asked to complete lesson 5/6/7/9 for HW2 and lesson 9 can be a little bit harder.

- We can go through parts of it together at the end of this lab (if we have time).
- You can always go to our office hour for help.

Basic matrix computation

- Addition
- Scalar multiplication
- Transpose
- Matrix multiplication
- Inverse
- Solving linear equations

Addition

Let $\mathbb{A} = [a_{ij}]_{n \times p}$ and $\mathbb{B} = [b_{ij}]_{n \times p}$, then $\mathbb{A} + \mathbb{B} = [a_{ij} + b_{ij}]_{n \times p}$
For example,

$$\mathbb{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

and

$$\mathbb{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Then

$$\mathbb{A} + \mathbb{B} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

Addition

```
# generate matrix A and B
A = matrix(c(3,-2,-1,4,1,2),nrow=2);A
```

```
##      [,1] [,2] [,3]
## [1,]    3   -1    1
## [2,]   -2    4    2
```

```
B = matrix(1:6,nrow=2);B
```

```
##      [,1] [,2] [,3]
## [1,]    1    3    5
## [2,]    2    4    6
```

```
A + B
```

```
##      [,1] [,2] [,3]
## [1,]    4    2    6
## [2,]    0    8    8
```

Scalar multiplication

Let $\mathbb{A} = [a_{ij}]_{n \times p}$, s be a scalar, then $s\mathbb{A} = [sa_{ij}]_{n \times p}$.
For example,

$$\mathbb{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then

$$s\mathbb{A} = \begin{bmatrix} sa_{11} & sa_{12} \\ sa_{21} & sa_{22} \end{bmatrix}$$

Scalar multiplication

```
# Use same matrix A
A
```

```
##      [,1] [,2] [,3]
## [1,]    3   -1    1
## [2,]   -2    4    2
```

```
# 5 time A
5 * A
```

```
##      [,1] [,2] [,3]
## [1,]   15   -5    5
## [2,]  -10   20   10
```

Transpose

Let $\mathbb{A} = [a_{ij}]_{n \times p}$, then $\mathbb{A}^\top = [a_{ji}]_{p \times n}$
For example,

$$\mathbb{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then

$$\mathbb{A}^\top = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

Transpose

```r
# Recall we have matrix A
A
```

```
##      [,1] [,2] [,3]
## [1,]    3   -1    1
## [2,]   -2    4    2
```

```r
# A transpose
C = t(A);C
```

```
##      [,1] [,2]
## [1,]    3   -2
## [2,]   -1    4
## [3,]    1    2
```

Matrix multiplication

Let $\mathbb{A} = [a_{ij}]_{n \times p}$ and $\mathbb{B} = [b_{ij}]_{p \times q}$, then $\mathbb{A}\mathbb{B} = [c_{ij}]_{n \times q}$ where $c_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj}$
For example,

$$\mathbb{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

and

$$\mathbb{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Then

$$\mathbb{A}\mathbb{B} = \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{bmatrix}$$

Matrix multiplication

```
# Recall we have matrix B and C
B
```

```
##      [,1] [,2] [,3]
## [1,]    1    3    5
## [2,]    2    4    6
```

```
C
```

```
##      [,1] [,2]
## [1,]    3   -2
## [2,]   -1    4
## [3,]    1    2
```

```
# Let's caculate BC by hand
```

Matrix multiplication

```
# Check with R
B %*% C
```

```
##      [,1] [,2]
## [1,]    5   20
## [2,]    8   24
```

```
# notice that matrix multiplication is not commutative
C %*% B
```

```
##      [,1] [,2] [,3]
## [1,]   -1    1    3
## [2,]    7   13   19
## [3,]    5   11   17
```

Inverse

$$\mathbb{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then

$$\mathbb{A}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$det(\mathbb{A}) = a_{11}a_{22} - a_{12}a_{21}$ is called the determinant of $\mathbb{A}$.
Need $det(\mathbb{A}) \neq 0$ for $\mathbb{A}$ to be invertible.
We only need to caculate the inverse a 2 by 2 matrix by hand

Inverse

```
# We can inverse higher dimensional matrix using R
# Produce a 3 by 3 matrix
set.seed(2018)
D = matrix(rnorm(9), nrow=3);D
```

```
##             [,1]       [,2]       [,3]
## [1,] -0.42298398  0.2708814  2.0994707
## [2,] -1.54987816  1.7352837  0.8633512
## [3,] -0.06442932 -0.2647112 -0.6105871
```

```
# Inverse of D
solve(D)
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.7065356 -0.3318892 -2.8986651
## [2,] -0.8518872  0.3345923 -2.4560648
## [3,]  0.4438772 -0.1100366 -0.2671086
```

Solving linear equation

Suppose we want to solve $\mathbb{A}\mathbf{x} = \mathbf{b}$, then $\mathbf{x} = \mathbb{A}^{-1}\mathbf{b}$. (Assuming $\mathbb{A}$ is invertible)
As a example question, we want to solving the following linear equations in R

$$\begin{array}{ccccccc} x & + & y & & & = & 2 \\ x & + & 2y & + & z & = & 1 \\ 3x & & & + & 2z & = & -3 \end{array}$$

```
# This is the A we what
A = matrix(c(1,1,3,1,2,0,0,1,2),nrow=3);A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    0
## [2,]    1    2    1
## [3,]    3    0    2
```

Solving linear equations

```r
# This is the b we what
b = c(2,1,-3)

# solve for x
x = solve(A) %*% b;x
```

```
##       [,1]
## [1,]  0.6
## [2,]  1.4
## [3,] -2.4
```

In lab activity

```
# We generate the data similar as the homework
randomMatrix <- function(p,q,values=-4:4){
  matrix(sample(values,size=p*q,replace=TRUE),p,q)
}
set.seed(2018)
A <- randomMatrix(2,2)
B <- randomMatrix(2,2)
```

1. Caculate the following by hand and check your results with R

- $\mathbb{A} + \mathbb{B}$
- $\mathbb{A}\mathbb{B}$
- $\mathbb{A}^{-1}$

2. Solve the following linear equations with R

$$\begin{array}{rcrcrcr} -x & + & y & + & z & = & 1.5 \\ x & + & 2y & - & z & = & -2 \\ 3x & & & + & 2z & = & -3 \end{array}$$

Lab ticket

1. Suppose $\mathbb{A}$ is a 4×6 matrix and $\mathbb{B}$ is a 3×6 matrix. What is the dimension of $\mathbb{A}\mathbb{B}^\top$?
2. Let

$$\mathbb{A} = \begin{bmatrix} 1 & 3 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

and

$$\mathbb{B} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 2 & 1 \end{bmatrix}.$$

Caculate $2\mathbb{A}\mathbb{B}$ by hand.