Midterm 2

Math/Stats 425, Winter 2013
Instructor: Edward Ionides

Name: ______________________________ UMID #: ______________________________

- **There are 6 questions, each worth 10 points.**
- **Points will be awarded for a clearly explained and accurate method, as well as for finding the correct answer.**
- **You are allowed to bring along to the test a single-sided sheet of notes.**
- **You are not allowed to use a calculator, or any other electronic device, during the exam. Electronic devices brought into the room should remain in a closed bag on the floor, and penalties will be applied if this rule is violated. For example, no cell phone usage for the duration of the exam, please!**

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1. A salesperson travels from house to house, selling subscriptions to a magazine. At each house he makes a successful sale with probability 0.05. Suppose it take 2 minutes to visit each house. The salesperson continues working each day until he has sold 10 subscriptions. Find expressions for the mean, variance and standard deviation of the amount of time the salesperson works each day. You may use properties of standard distributions without proof. Comment on any assumptions your answer requires that are not explicitly given in the question.

2. The average number of patients arriving at a large hospital with a broken limb is 5 per day if there is a snow fall, and 3 per day otherwise. During winter, the chance of snow falling on any particular day is 0.2. Suppose that no patients with broken limbs arrive on a particular day. What is the chance that snow fell on this day? Explain your assumptions.

3. Let $Y = X^2$, where X is a continuous random variable with density

$$f(x) = \begin{cases} (1/18)\sqrt{x} & 0 \le x \le 9 \\ 0 & x < 0 \text{ or } x > 9 \end{cases}$$

(a) Find $\mathbb{E}[Y]$.

(b) Find the probability density function of Y. Hint: it may help you to first calculate the cumulative distribution function of Y.

4. Let X be an exponential random variable, taking values on the positive real line with density

$$f(x) = \lambda\, e^{-\lambda x}.$$

Show that X has the memoryless property, that for any positive real numbers $a < b$,

$$\mathbb{P}(X > b \,|\, X > a) = \mathbb{P}(X > b - a).$$

5. A fair six-sided die is rolled three times. Let X be the smallest value of the three rolls, e.g. if the rolls are 2, 5 and 2 then X takes the value 2. Find an expression for the cumulative distribution function of X.

6. Let X have a Poisson(λ) distribution, with probability mass function $p(k) = \frac{\lambda^k}{k!}e^{-\lambda}$, and let $Y = 2^{-X}$. Find the expected value of Y.