Sample final exam

Math/Stats 425 (Instructor: Edward Ionides)

Name: _____ UMID #: _____

- **There are 7 questions, each worth 10 points.**
- **Points will be awarded for a clearly explained and accurate method, as well as for finding the correct answer.**
- **You are allowed to bring along to the test a single-sided sheet of notes.**
- **You are not allowed to use a calculator, or any other electronic device, during the exam. Electronic devices brought into the room should remain in a closed bag on the floor, and penalties will be applied if this rule is violated. For example, no cell phone usage for the duration of the exam, please!**

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

1. Let X be a continuous random variable, with

$$P(X > x) = (1 - x)^2, \qquad 0 \leq x \leq 1.$$

(i) Find the cumulative distribution function of X.

(ii) Find the probability density function of X.

(iii) Find the expected value of X.

2. One evening, Fyodor decides to play 10 games of roulette, betting \$100 on black each time (this bet wins \$100 with probability $\frac{18}{38}$, and otherwise loses \$100).

   (i) Find the expected value and standard deviation of his total winnings. You should write an expression which is suitable for evaluation, but you are not asked to evaluate it.

   (ii) Find an expression for the exact chance (*i.e.*, not using a normal approximation) that he loses \$400 or more over the course of the evening.

3. Anne and Bin both plan to take the 16.10 train to Chicago. Anne's arrival at the train station is uniformly distributed between 0 and 20 minutes before the train departs. Bin's arrival is uniformly distributed between 0 and 15 minutes before the train departs. Find the chance that Anne arrives at the station before Bin. You may suppose that their arrivals are independent.

4. 24% of college freshmen and 16% of seniors regularly drink to excess. Find, approximately the chance that in a randomly selected sample of 200 freshmen and 200 seniors there are more freshmen who reguarly drink to excess. Write your answer in terms of the standard normal cumulative distribution function, $\Phi(x)$.

5. A Math course has a sequence of three exams (Midterm 1, Midterm 2, Final). The chance of a student failing an exam is 1/20, unless that student failed at least one exam earlier in the course, in which case the chance of failing goes up to 1/3. Given that a student fails the final, find an expression for the chance that the student failed at least one of the two midterms.

6. Suppose X has Poisson(λ) distribution, so $\mathbb{P}(X = k) = \lambda^k e^{-\lambda}/k!$ for $k = 0, 1, \ldots$. Suppose Y is independent of X and has Poisson(μ) distribution. Let $Z = X + Y$. Find the joint probability mass function of X and Z.

7. Ten balls are dropped at random into two urns, A and B. Each ball independently falls into urn A with probability $1/2$ and urn B with probability $1/2$. Let X be the number of balls falling into urn A, and Y the number falling into B.

   (i) Find the variance of X.

   (ii) Find the variance of $X + Y$.

   (iii) Find the covariance of X and Y. Hint: it may help you to use parts (i) and (ii).