

Sample Midterm 2

Math/Stats 425 (Instructor: Edward Ionides)

Name: _____ UMID #: _____

- There are 6 questions, each worth 10 points.
- Points will be awarded for a clearly explained and accurate method, as well as for finding the correct answer.
- You are allowed to bring along to the test a single-sided sheet of notes.
- You are not allowed to use a calculator, or any other electronic device, during the exam. Electronic devices brought into the room should remain in a closed bag on the floor, and penalties will be applied if this rule is violated. For example, no cell phone usage for the duration of the exam, please!

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1. Suppose X has probability mass function $p(k) = k/10$ for $k = 1, 2, 3, 4$. Plot the probability mass function of $X^2 - 4X + 3$.

2. There are 4 balls in a box, numbered 1, 2, 3 and 4. I invite you to draw two balls, at random without replacement. I will then pay you an amount in dollars equal to the sum of the numbers on the two balls you picked. Calculate the expected value and variance of your winnings from playing. What would be the fair price you should pay to play? Explain.

3. Use a Poisson approximation to write an expression approximating the chance that in a university of 30,000 students there are exactly k students who share the same birthday as both his/her mother and father. Explain your reasoning.

4. If it is raining, students come to class on Friday independently with probability 0.8. If it is not raining this probability increases to 0.9. The chance of rain this Friday is 0.3. Find an expression for the chance that at least 19 students will show up to class on Friday out of a class of 20 students.

5. Suppose X has Geometric ($1/3$) distribution (i.e., X has probability mass function $p(k) = (1/3)(2/3)^{k-1}$, for $k = 1, 2, \dots$). Find the expected value of e^{-X} . Explain your reasoning.

6. Suppose X and Y are continuous random variables whose probability density functions and cumulative distribution functions are $f_X(\cdot)$, $f_Y(\cdot)$, $F_X(\cdot)$ and $F_Y(\cdot)$ respectively. Z is a random variable defined as follows: a biased coin is flipped, which lands on heads with probability p . If the coin lands on heads then Z is assigned to take the same value as X , otherwise Z is assigned to take the same value as Y . Find expressions for the probability density function and cumulative distribution function of Z . Explain your reasoning.