Homework 1 (Math/Stats 425, Winter 2013)

Due Tuesday Jan 22, in class

1. A, B and C take turns flipping a coin. The first one to get a head wins. Define the sample space for this experiment as

$$\mathbb{S} = \begin{cases} 1, 01, 001, 0001, \ldots \\ 000 \cdots \end{cases}$$

(a) Describe this sample space in words.

(b) Let $E = \{A \text{ wins}\}$ and $F = \{B \text{ wins}\}$. Define the following events formally, as subsets of the sample space $\mathbb{S}$:

(i) E.

(ii) F.

(iii) $(E \cup F)^c$.

2. A town with a population of 100,000 has three newspapers: I, II and III. The proportions of the population reading the papers are as follows:

I: 10 percent	I and II: 8 percent
II: 30 percent	I and III: 2 percent
III: 5 percent	II and III: 4 percent
I and II and III: 1 percent	

(a) Find the number of people who read only one newspaper.

(b) How many people read at least two newspapers?

(c) If I and III are morning papers and II is an evening paper, how many people read at least one morning paper plus an evening paper?

(d) How many people do not read any newspapers?

(e) How many people read only one morning paper and one evening paper?

3. A small community organization consists of 20 families, of which 4 have one child, 8 have two children, 5 have three children, 2 have four children and 1 has five children.

(a) If one of these families is chosen at random, what is the probability it has i children, for $i = 1, 2, \ldots, 5$?

(b) If one of the children is randomly chosen, what is the probability this child comes from a family having i children, for $i = 1, 2, \ldots, 5$?

(c) Give a brief verbal description of these results, explaining of the differences between the two sets of probabilities.

4. If two dice are rolled, find the probability that the sum of their upturned faces equals i, for each $i = 2, 3, \ldots, 12$. Comment briefly on the assumptions you have made to carry out your computation.

5. Let E and F be two events in a sample space $\mathbb{S}$, with $\mathbb{P}(E) = 0.9$ and $\mathbb{P}(F) = 0.8$.

   (i) Argue that $\mathbb{P}(E \cap F) \geq 0.7$.

   (ii) In general, prove Bonferroni's inequality, namely,

$$\mathbb{P}(E \cap F) \geq \mathbb{P}(E) + \mathbb{P}(F) - 1.$$

   Instructions: Use a less formal approach for (i) than for (ii); otherwise, (i) becomes trivial as a special case of (ii). For (ii), a *proof* means an argument which explains the correctness of the result to your own satisfaction. A good proof will also explain the correctness of the result clearly to other people.

Recommended reading:
Sections 2.1–2.5 and 2.7 in Ross "A First Course in Probability," 8th edition.

There are too many examples to study them all carefully! Some suggested examples from Chapter 2 are 3a, 3b, 4a, 7a. The examples involving counting problems will be appropriate after we have covered the material in Chapter 1.

Supplementary exercises: The self-test exercises on pages 56–57 are optional, but recommended. Do not turn in solutions—they are in the back of the book. #1 and #2 are similar to the homework. #14 is in the same spirit as homework question 5. You may want to return to these exercises when we've finished studying counting methods.