

Homework 2 (Math/Stats 425, Winter 2013)

Due Tuesday Jan 29, in class

1. A child has 12 blocks, of which 6 are black, 4 are red, 1 is white and 1 is blue. If the child puts the blocks in a line, how many arrangements are possible?

Note: there is some ambiguity in this question. If you can see more than one way it can be interpreted, you should comment briefly on this and follow what you think is the most reasonable interpretation.

2. In how many ways can 3 novels, 2 mathematics books, and 1 chemistry book be arranged on a bookshelf if

(a) the books can be arranged in any order;

(b) the mathematics books must be together and the novels must be together; (c) the novels must be together but the other books can be arranged in any order?

3. Poker dice is played by simultaneously rolling 5 dice. Show that

(a) $\mathbb{P}\{\text{no two alike}\} = 0.0926$;

(b) $\mathbb{P}\{\text{one pair}\} = 0.4630$;

(c) $\mathbb{P}\{\text{two pairs}\} = 0.2315$;

(d) $\mathbb{P}\{\text{three alike}\} = 0.1543$;

(e) $\mathbb{P}\{\text{full house}\} = 0.0386$;

(f) $\mathbb{P}\{\text{four alike}\} = 0.0193$;

(g) $\mathbb{P}\{\text{five alike}\} = 0.0008$;

Note: a full house is three of a kind together with a pair.

4. A forest contains 20 elk, of which 5 are captured, tagged and then released. A certain time later 4 of the elk are captured. What is the probability that exactly 2 of these 4 have been tagged? Comment briefly on the assumptions you are making and their reasonableness (this kind of capture-recapture study, together with the assumptions required to do a probabilistic analysis of the data, is a fundamental procedure for ecological studies).

5. Five people, designated A, B, C, D, E , are arranged in a line. Assuming that each order is equally likely, find the chance that

(a) there is exactly one person between A and B ;

(b) there are exactly two people between A and B ;

(c) there are three people between A and B .

6. Given 20 people, what is the probability that among the 12 months of the year there are 4 months containing exactly 2 birthdays and 4 months containing exactly 3 birthdays? Comment briefly on your assumptions.

Recommended reading:

Chapter 1 (we will not cover Section 1.6, on the number of integer solutions to equations).

Supplementary exercises: The self-test problems at the end of Chapter 1 and those for Chapter 2 involving counting problems are both relevant.