

Homework 3 (Math/Stats 425, Winter 2013)

Due THURSDAY FEB 7, in class

1. Suppose that 5 percent of men and 0.25 percent of women are colorblind. A colorblind person is chosen at random. What is the probability of this person being male?
2. A red die, a blue die, and a yellow die (all six sided) are rolled. Let R , B , and Y be the respective numbers showing on the dice.

(a) Find the probability no two dice land on the same number, i.e.,

$$\mathbb{P}(\{R \neq B\} \cap \{R \neq Y\} \cap \{B \neq Y\}).$$

(b) Given that no two dice land on the same number, find the conditional probability that the blue die shows less than the yellow die which shows less than the red die? i.e.,

$$\mathbb{P}(B < Y < R \mid \{R \neq B\} \cap \{R \neq Y\} \cap \{B \neq Y\}).$$

(c) Find $\mathbb{P}(B < Y < R)$.

3. The color of a person's eyes is determined by a single pair of genes. If they are both blue-eyed genes, then the person will have blue eyes; if they are both brown-eyed, or if one is blue-eyed and the other brown-eyed, then the person will have brown eyes. The brown-eyed gene is therefore said to be *dominant* over the blue-eyed gene. A newborn child independently receives one eye gene from each of its parents, chosen at random from the two eye genes belonging to each parent. Suppose that Smith and both of his parents have brown eyes, but Smith's sister has blue eyes.

(a) What is the probability that Smith possesses a blue-eyed gene?

Now suppose that Smith's wife has blue eyes.

(b) What is the probability that their first child will have blue eyes?

(c) If their first child has brown eyes, what is the probability that their next child will also have brown eyes?

4. If $A \subset B$, express the following probabilities as simply as possible:

(a) $\mathbb{P}(A \mid B)$; (b) $\mathbb{P}(A \mid B^c)$; (c) $\mathbb{P}(B \mid A)$; (d) $\mathbb{P}(B \mid A^c)$.

5. Independent trials that result in a success with probability p are successively performed until a total of r successes is obtained. Show that the probability that exactly n trials are required is

$$\binom{n-1}{r-1} p^r (1-p)^{n-r}.$$

Recommended reading:

Chapter 3. There are too many examples to study them all carefully, but you could select some whose topic interests you. The class notes, homework, and practice exam (to be posted soon) are more directly relevant to the upcoming midterm (in class on February 12).