Homework 5 (Math/Stats 425, Winter 2013)

Due Tuesday March 12, in class

1. A newsboy purchases papers at 10 cents and sells them at 15 cents. However, he is not allowed to return unsold papers. Suppose his daily demand is a binomial random variable with $n = 10$, $p = 1/3$.

   (a) How many papers should he purchase to maximize his expected profit?

   Hint: it is not enough to show that the newsboy is most likely to have 3 customers. The solution has to depend on the price that the newsboy buys and sells papers at. For example, make sure that your method would also work if the question is changed so that the boy buys papers at 5 cents and sells them at 15.

   (b) Comment briefly on the use of the binomial distribution as a probability model in this situation.

2. A student is getting ready to take an important oral examination and is concerned about the possibility of having an 'on' day or an 'off' day. He figures that if he has an on day, then each of his examiners will pass him independently of each other with probability 0.8, whereas, if he has an off day, this probability will be reduced to 0.4. Suppose that the student will pass the examination if a majority of the examiners pass him. If the student feels that he is twice as likely to have an on day as an off day, should he request an examination with 3 examiners or with 5 examiners?

3. A certain typing agency employs two typist. The average number of errors per article is 3 when typed by the first typist and 4.2 when typed by the second. You read an article carefully and find it has 6 errors. Estimate the chance that it was typed by the second typist. Explain your reasoning.

4. Let X be a binomial random variable with parameters n and p. Show that

$$\mathbb{E}\left[\frac{1}{X+1}\right] = \frac{1-(1-p)^{n+1}}{(n+1)p}.$$

5. Let X be a binomial random variable with parameters n and p. What value of p maximizes $\mathbb{P}(X = k)$ for a given value of k in $\{0, 1, \ldots, n\}$?

   This is an example of a statistical method to estimate an unknown parameter, called *maximum likelihood estimation*. If the number of trials, n, is known and the observed number of successful trials is k, then we might want to follow this approach to estimate the unknown success probability.

6. Let X be a Poisson random variable with parameter λ. Find the value of λ which maximizes $\mathbb{P}(X = k)$ for a given non-negative integer k.

   This is another example of maximum likelihood estimation, where we assume that the data (i.e., the number k) was the outcome of a Poisson random variable. In this case, we want to estimate the unknown parameter λ.

Recommended reading:
Sections 4.6–4.8 in Ross "A First Course in Probability," 8th edition. This course will not include the Zeta distribution (Sec. 4.8.4).