Homework 7 (Math/Stats 425, Winter 2013)

Due Tuesday April 9, in class

1. The annual rainfall (in inches) in a certain region is modeled as being normally distributed with $\mu = 40$ and $\sigma = 4$. According to this model, what is the probability that it will take over 10 years before a year occurs having rainfall above 50 inches? What assumptions are you making?

2. The number of years a new radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio, what is the probability that it will be working after an additional 8 years? Comment on your assumptions.

3. Find the density function of $R = a\sin(\Theta)$, where a is a fixed constant and Θ is uniformly distributed on $(-\pi/2, \pi/2)$.

   Note: such a random variable R arises in the theory of ballistics. If a projectile is fired from the origin at an angle α from the earth with speed ν, then the point R at which it returns to the earth can be expressed as $R = (\nu^2/g)\sin(2\alpha)$ where g is the gravitational constant.

4. Suppose that 3 balls are chosen successively, without replacement, from an urn containing 5 white balls and 8 red balls. Let X_i equal 1 if the ith ball drawn is white, and otherwise X_i equals 0. Write the joint probability mass function of

   (a) X_1 and X_2

   (b) X_1, X_2 and X_3

   (c) $X_1 + X_2$ and $X_1 + X_3$

5. The joint probability density function of X and Y is given by

$$f(x, y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) \quad 0 < x < 1,\, 0 < y < 2$$

   (a) Verify that this is indeed a valid joint density function.

   (b) Compute the marginal density function of X.

   (c) Find $\mathbb{P}(X > Y)$.

   (d) Find $\mathbb{P}(Y > 1/2 \,|\, X < 1/2)$.

   (e) Find $\mathbb{E}[X]$.

   (f) Find $\mathbb{E}[Y]$

Recommended reading:
Sections 5.4, 5.5, 5.7 and 6.1 in Ross "A First Course in Probability," 8th edition. This course will not include the material in Section 5.6.