Homework 9 (Math/Stats 425, Winter 2013)

Due Tuesday April 23, in class

1. The joint probability mass function of X and Y is given by

$$\begin{array}{rclcrcl} p(1,1) & = & 1/8 & \qquad & p(1,2) & = & 1/4 \\ p(2,1) & = & 1/8 & & p(2,2) & = & 1/2 \end{array}$$

   (a) Compute the conditional mass function of X given $Y = i$ for $i \in \{1, 2\}$.

   (b) Are X and Y independent?

   (c) Compute $\mathbb{P}(XY < 3)$, $\mathbb{P}(X + Y > 2)$, and $\mathbb{P}(X/Y > 1)$.

2. The joint density function of X and Y is given by

$$f(x, y) = x\, e^{-x(y+1)}, \quad x > 0,\ y > 0.$$

   Find the conditional density of X given $Y = y$ and the conditional density of Y given $X = x$.

3. N people arrive separately to a professional dinner. Upon arrival, each person looks to see if he or she has any friends among those present. That person then either sits at the table of a friend or at an unoccupied table if none of those present is a friend. Assuming that each of the $\binom{N}{2}$ pairs of people are, independently, friends with probability p, find the expected number of occupied tables.

   Hint: you might want to consider indicator random variables for the event that the ith arrival sits at a previously unoccupied table.

4. A total of n balls, numbered 1 through n, are put into n urns, also numbered 1 through n. Each ball is placed independently, with ball i equally likely to go into any of the urns numbered $1, 2, \ldots, i$. Find

   (a) the expected number of urns that are empty;

   (b) the probability that none of the urns is empty.

5. If X and Y are independent and identically distributed with mean μ and variance σ^2, find

$$\mathbb{E}\big[(X - Y)^2\big].$$

6. A group of 20 people—consisting of 10 men and 10 women—are randomly arranged into 10 pairs of 2 each.

   (a) Compute the expectation and the variance of the number of pairs that consist of a man and a woman.

   (b) Now suppose the 20 people consisted of 10 married couples. Compute the mean and variance of the number of married couples that are paired together.

Recommended reading:
Sections 7.1, 7.2, 7.3, 7.4 in Ross "A First Course in Probability," 8th edition.