

Problem solving strategies

1. Define notation

- formalize as a math problem.

2. Pick an approach:

- Sample-space approach:

Reasoning from the axioms; Venn diagrams; counting methods; equally likely outcomes; think of \cap as “intersection,” \cup as “union,” etc.

- Higher-level techniques:

Bayes’ rule; law of total probability; tree diagrams; the multiplication rule; think of “mutually exclusive” rather than “disjoint”; think of \cap as “and,” \cup as “or,” etc.

Example 1. Cards are dealt, one at a time, from the top of a shuffled deck of 52 cards. What is the chance that the card immediately following the first Ace is the Ace of Spades?

Solution.

Example 2. A fair die is thrown repeatedly.
Find the chance of rolling two sixes before rolling
a one.

Solution.

Example 3: Craps. A player rolls 2 dice. Let the sum be K . If $K = 2, 3, 12$ the player loses. If $K = 7, 11$ she wins. Otherwise, she keeps rolling and wins if the total K reoccurs before a sum of 7. Find the chance she wins.

Solution.

Example 4: Tennis tournament.

Eight evenly matched players play a knock-out tennis tournament with a random draw. Find the chance that Angus plays Bill at some point of the tournament.

Solution.

Example 5. X and Y play a series of independent games. X wins each with probability p , and Y wins with probability $1 - p$. They stop when the number of wins of one player is two greater than that of the other player. The player winning more games is the match winner.

(a) Find the chance that exactly four games are played.

(b) Find the chance that A wins the match.