

Statistics 620  
Midterm exam, Fall 2012

Name: \_\_\_\_\_ UMID #: \_\_\_\_\_

**Midterm Exam**

- There are 4 questions, each worth 10 points.
- You are allowed a single-sided sheet of notes.
- You are not allowed to make use of a calculator, or any other electronic device, during the exam.
- Credit will be given for clear explanation and justification, as well as for getting the correct answer.
- Cross out any working that you do not wish to be considered as part of your solution. You are advised not to erase unfinished working since partial credit may be available for an indication that an appropriate method was attempted, even if it was later rejected.
- You may continue your solutions on the reverse side of the pages. Additional paper is available, should you require it.

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. Workplace accidents occur at University of Michigan as a Poisson process with rate  $\lambda$ . Each accident independently results in a lawsuit with probability  $p$ . Let  $T$  denote the time of the first lawsuit, and let  $A$  be the total number of accidents in the random time interval  $[0, T]$ . Find  $P[A = n | T = t]$ .

Hint: you may (if you wish) use without proof the result that, if  $N(t)$  is a Poisson process and each event is independently classified as Type I or Type II, then the corresponding counting processes  $N_1(t)$  and  $N_2(t)$  for events of each type are independent Poisson processes.

**2.** Let  $\{N(t), t \geq 0\}$  be a renewal process, with corresponding arrival times  $\{S_n\}$  and inter-arrival times given by  $X_n = S_n - S_{n-1}$ . Suppose  $X_n$  has distribution  $F$ . Define the age process by  $A(t) = t - S_{N(t)}$ , namely the time since the most recent arrival.

(i) Show that  $E[A(t)] = t\bar{F}(t) + \int_0^t (t-u)\bar{F}(t-u)dm(u)$ .

Hint: recall that  $dF_{S_{N(t)}}(u) = \begin{cases} \bar{F}(t) + \bar{F}(t)dm(0) & \text{for } u = 0 \\ \bar{F}(t-u)dm(u) & \text{for } u > 0 \end{cases}$

(ii) Hence, show that  $\lim_{t \rightarrow \infty} E[A(t)] = E[X^2]/2E[X]$ , where  $X$  has distribution  $F$ .

**3.** Trials are performed in sequence. If the two most recent previous trials were both successes, the next trial is a success with probability 0.8; otherwise, the chance of success is 0.5.

(i) Define state 1 to be {most recent trial was a failure}, state 2 to be {most recent trial was a success, and the preceding trial was a failure} and state 3 to be {last two trials were successes}. Let  $X_n$  be the state after the  $n$ th trial. Explain why  $\{X_n\}$  is a Markov chain, and find the transition matrix  $P = [P_{ij}]$ .

(ii) Use the Markov chain in (i) to find the long run proportion of trials that are successes.

4. If an individual has never had a previous automobile accident, then the probability he or she has an accident in the next  $h$  time units is  $\beta h + o(h)$ ; on the other hand, if she or he has ever had a previous accident, then the probability is  $\alpha h + o(h)$ . Find the expected number of accidents an individual has by time  $t$ .

Hint: you may like to condition on the time of the first accident.