Statistics 620
Midterm exam, Fall 2013

Name: ________________________ **UMID #:** ________________________

Midterm Exam

- **There are 4 questions, each worth 10 points.**
- **You are allowed a single-sided sheet of notes.**
- **You are not allowed to make use of a calculator, or any other electronic device, during the exam.**
- **Credit will be given for clear explanation and justification, as well as for getting the correct answer.**
- **Cross out any working that you do not wish to be considered as part of your solution. You are advised not to erase unfinished working since partial credit may be available for an indication that an appropriate method was attempted, even if it was later rejected.**
- **You may continue your solutions on the reverse side of the pages. Additional paper is available, should you require it.**

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. Richard catches trout according to a Poisson process with rate 0.1 minute^{-1}. Suppose that the trout weigh an average of 4 pounds with a standard deviation of 2 pounds. Find expressions for the mean and standard deviation of the total weight of fish he catches in two hours.

2. A light bulb has a lifetime that is exponential with a mean of $\mu = 200$ days. When it burns out, a janitor replaces it immediately. In addition, there is a handyman who comes, on average, $h = 3$ times per year according to a Poisson process and replaces the lightbulb as "preventative maintenance."

(a) Find the average time between bulb replacements, in terms of μ and h.

(b) In the long run, what fraction of the replacements are due to failure? Your answer should be an expression in terms of μ and h.

3. Let $\{N(t), t \geq 0\}$ be a renewal process, with corresponding arrival times $\{S_n\}$ and inter-arrival times $X_n = S_n - S_{n-1}$ with $E[X_n] = \mu$. Show that $\lim_{t\to\infty} N(t)/t = 1/\mu$, with probability one.

Note: you can use without proof the strong law of large numbers for S_n.

4. Consider the following approach to shuffling a deck of n cards. Starting with any initial ordering of the cards, one of the numbers $1, 2, \ldots, 52$ is chosen at random and with equal probability. If number i is chosen, we move the card from position i in the deck to the top, i.e. to position 1. We repeatedly perform the same operation. Show that, in the limit, the deck is perfectly shuffled in the sense that the resultant ordering is equally likely to be any of the $n!$ possible orderings.