

# STATS 700-002 Class 2.

## The coalescent

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September 4, 2025

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Kingman, J. F. C. (1982). The coalescent. *Stochastic Processes and their Applications*, 13(3), 235-248.

[https://doi.org/10.1016/0304-4149\(82\)90011-4](https://doi.org/10.1016/0304-4149(82)90011-4).

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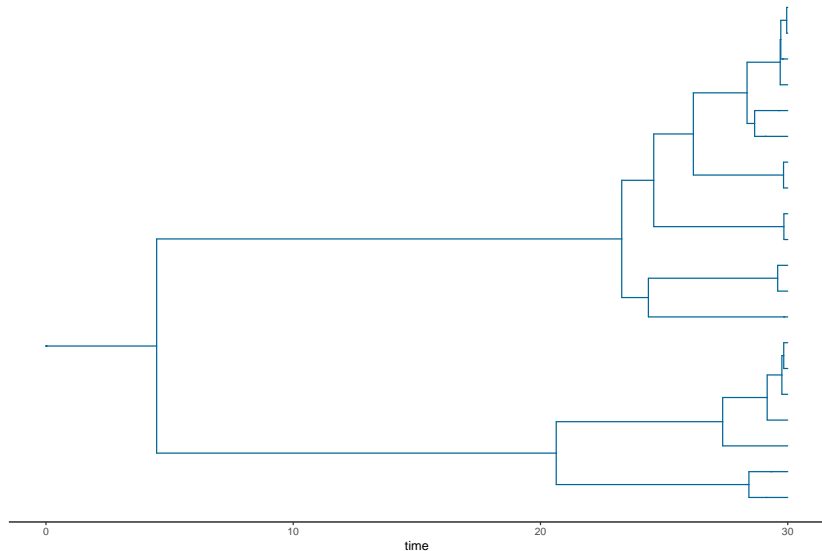
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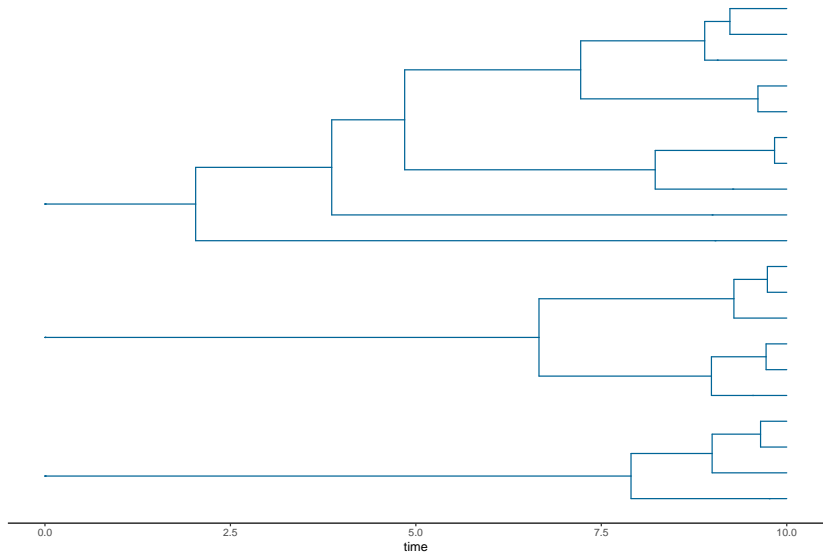
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- ▶ In terms of genealogical trees, what does the *transit time*  $T$  introduced in (1.11) correspond to?



# Coalescent trees





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## Theorems 3 and 4

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- ▶ What extensions can you envision?

## Section 5

- ▶ What is the relationship between the paintbox construction and a genealogical tree?

# Bibliography I

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